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FLOW OF A GAS OUT OF THE VESSEL TO A MEDIUM WITH COUNTERPRESSURE UNDER THE CONDITIONS OF INTENSE HEAT SUPPLY

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The basic regularities of the influence of heat supply on the parameters of the flow have been elucidated with the use of analytical and numerical solutions of the quasi-one-dimensional problem on flow, through a pipe, of the gas out of a vessel with prescribed pressure and stagnation temperature to a medium with a prescribed counterpressure. It has been shown that changeover from the stationary outflow to an unsteady self-oscillatory regime of outflow is possible with increase in the thermal action on the flow. The asymptotic dependences of the flow parameter in the case of an infinitely large supplied heat have been obtained; these dependences are used in finding the scale quantities and unified generalized dependences for the period-averaged parameters of nonstationary self-oscillatory flows.

Introduction. The influence of heat supply on the parameters of outflow of a gas through a pipe has been considered in many works, e.g., in [1–3]. For determination of the velocity and pressure in the outlet cross section of a pipe or a channel, Abramovich et al. have obtained the following relations:

$$\frac{u_2}{u_1} = \frac{1}{2}\left(1 + \frac{1}{\lambda_1^2}\right) \pm \sqrt{\frac{1}{4}\left(1 - \frac{1}{\lambda_1^2}\right)^2 - \frac{q}{\lambda_1^2}}\ , \tag{1}$$

$$\frac{p_2}{p_1} = 1 + \gamma M_1^2 \left(1 - \frac{u_2}{u_1}\right), \tag{2}$$

where $q = Q/(Gi_0)$.

These relations are usually used to elucidate the basic regularities of the influence of heat supply on the variation in the parameters along the pipe for prescribed parameters in the inlet cross section. However, under actual conditions, we have flow of a gas through a pipe out of a fairly large vessel (receiver), in which the pressure and temperature of the gas are prescribed, to the ambient medium in which the pressure is prescribed. Under these conditions, the parameters in the inlet cross section of the pipe are not prescribed and are to be determined from the difference of the pressures in the receiver and the ambient space and from the heat-supply intensity. The issues of the influence of heat supply on the parameters of outflow, in particular, on the gas flow rate, have not been studied for such flows.

We consider flow of a gas out of a vessel (receiver) with the parameters of a quiescent gas — pressure p_0 and temperature T_0 — to a medium with a prescribed pressure p_2 through a pipe (channel), in which heat supply distributed along the length and with a constant intensity is carried out. Here the coefficient of velocity λ_1 in the inlet pipe cross section is considered to be unknown. It is necessary to determine it so that the ratio of the pressure at the pipe inlet to the pressure in the receiver is equal to that prescribed, i.e., the relation

$$\frac{p_2}{p_0} = \frac{p_2}{p_1}\frac{p_1}{p_0} = \frac{p_2}{p_1}\left(1 - \frac{\gamma - 1}{\gamma + 1}\lambda_1^2\right)^{\frac{\gamma}{\gamma - 1}} . \tag{3}$$

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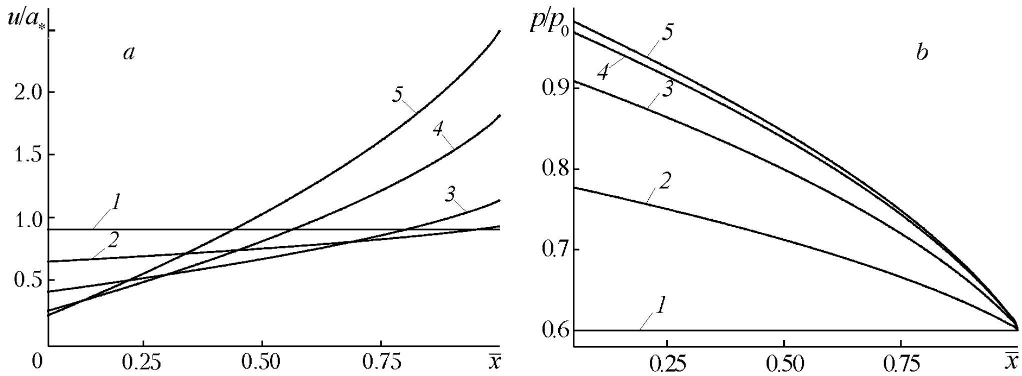

Fig. 1. Distribution of the dimensionless gas parameter along the pipe at $\delta p = 0.6$: a) velocity; b) pressure; 1) $q = 0$, 2) 0.2, 3) 1.1, 4) 5.0 and 5) 10.6

must hold.

Using (1) and (2) for the prescribed pressure ratio $\delta p = p_2/p_0$, from relation (3) we obtain a transcendental equation for λ, which is conveniently solved by the method of chords. Expressions (1) and (2) yield that, for prescribed parameters in the inlet pipe cross section, the supply of heat to a subsonic flow leads to an increase in the velocity and consequently to a decrease in the pressure at the pipe outlet. For the prescribed δp the supply of heat must lead to a decrease in the velocity and a growth in the gas pressure in the inlet pipe cross section. This is confirmed by the results of calculations (Fig. 1) carried out with the use of relations (1) and (2). This figure shows the distributions of the dimensionless velocity u/a_* and pressure p/p_0 along the pipe.

Analytical Evaluation of the Prerequisites for Occurrence of Self-Oscillations in Heat Supply. As follows from Fig. 1, when the heat supply is fairly large, the velocity coefficient in the inlet pipe cross section becomes much less than unity. In this case we may disregard λ_1^2 compared to unity, as a result of which we obtain, from (1) and (2),

$$\lambda_1^2 \frac{u_2}{u_1} = \frac{1}{2} - \sqrt{1 - 4\lambda_1^2 q}\,, \quad \delta p = 1 - \frac{2\gamma}{\gamma + 1} \lambda_1^2 \frac{u_2}{u_1}\,.$$

The solution of these equations can be written in the form

$$\lambda_1 = \sqrt{\frac{f(\delta p)\,(1 - f(\delta p))}{q}}\,, \tag{4}$$

$$\frac{u_2}{u_1} = \frac{q}{1 - f(\delta p)}\,, \tag{5}$$

where $f(\delta p) = \dfrac{\gamma + 1}{2\gamma}(1 - \delta p)$.

It is of interest to write an expression for the velocity coefficient in the outlet pipe cross section $\lambda_2 = \dfrac{u_2}{u_1}\dfrac{u_1}{a_{*1}}\dfrac{a_{*1}}{a_{*2}} = \dfrac{u_2}{u_1}\dfrac{\lambda_1}{\sqrt{1+q}}$. Taking account of (4) and (5), we obtain

$$\lambda_2 = \sqrt{\frac{f(\delta p)}{1 - f(\delta p)}} + O(\lambda_1^2)\,. \tag{6}$$

We obtain $\lambda_2 < 1$ for $f(\delta p) < 0.5$ or $\delta p > 1/(\gamma + 1)$; for $\gamma = 1.4$ we have $\delta p > 0.4167$. We note that in flow of the gas out of the vessel without heat supply, we have $\lambda_2 < 1$ for $\delta p > \left(\dfrac{2}{\gamma + 1}\right)^{\gamma/(\gamma - 1)}$; for $\gamma = 1.4$ the quantity δp is more than 0.5228.

The gas velocity in the outlet pipe cross section will be determined by the relation

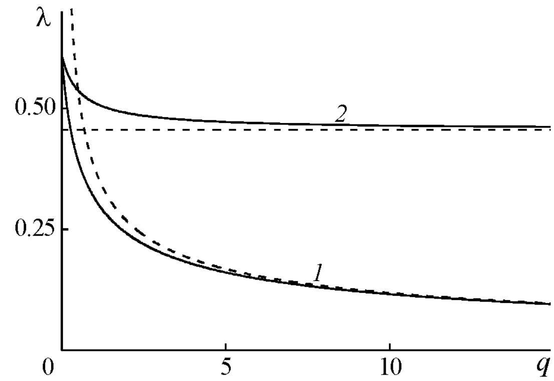

Fig. 2. Change in the velocity coefficient of the gas in the inlet (1) and outlet (2) cross sections of the pipe as a function of the supplied heat: solid curves, exact solutions; dashed curves, relations (4) and (6).

$$u_2 = \frac{u_2}{u_1}\lambda_1 a_{*1} = a_* \sqrt{\frac{f(\delta p)}{1-f(\delta p)}}\sqrt{q}\,, \tag{7}$$

where $a_* = \sqrt{\frac{2\gamma}{\gamma+1}RT_0}$ is the critical gas velocity in the receiver. The error of the above formulas is less than 4% for $q > 5$.

Thus, in intense supply of heat to a subsonic flow flowing out, through a pipe, to a medium with a prescribed counterpressure, the velocity coefficient in the inlet cross section λ_1 decreases in inverse proportion, whereas the velocity in the outlet cross section grows in direct proportion to the root squared of the quantity of heat q supplied to a unit mass of the outflowing gas. The velocity coefficient in the outlet cross section with an error of the order $\lambda_1^2 \sim 1/q$ is independent of the quantity of the supplied heat.

Figure 2 gives the dependences of the velocity coefficient in the inlet λ_1 and outlet λ_2 cross sections, which have been calculated from the asymptotic relations (4) and (6) and have been obtained by exact solution of the equations for $\delta p = 0.8$. It follows from these relations that the velocity coefficient in the inlet cross section tends to zero when $q \to \infty$, i.e., a paradoxical situation is created: the gas does not flow out of the receiver in the presence of low pressure in the ambient space. The way out of this situation is destruction of the stationary flow and transition to a self-oscillatory regime of outflow. Self-oscillations can occur in gas flow with a fairly high intensity of heat supply. Heat fluxes for which self-oscillations occur increase with difference in the pressure in the receiver and in the ambient medium.

Numerical Investigations of Self-Oscillatory Flows in a Pipe in Heat Supply. To find the parameters distributed over the pipe length and to investigate nonstationary processes, we use a quasi-one-dimensional flow model within whose framework the parameters variable over the pipe cross section are replaced by certain values constant over the cross section by means of averaging [3]. We write these equations in divergent form for dimensionless parameters

$$\frac{\partial\bar\rho}{\partial\tau} + \frac{\partial\bar\rho\bar u}{\partial\bar x} = 0\,,\quad \frac{\partial\bar\rho\bar u}{\partial\tau} + \frac{\partial(\bar\rho\bar u^2 + \bar p)}{\partial\bar x} = 0\,,\quad \frac{\partial\bar\rho\bar E}{\partial\tau} + \frac{\partial\bar\rho\bar u\bar i}{\partial\bar x} = \frac{1}{2}\frac{\gamma+1}{\gamma-1}\delta q\,, \tag{8}$$

where $\tau = ta_*/L$, $\bar x = x/L$, $\bar\rho = \rho/\rho_0$, $\bar u = u/a_*$, $\bar p = p/(\rho_0 a_*^2)$, $\bar E = E/a_*^2$, $\bar i = i/a_*^2$, $E = \frac{1}{\gamma-1}\frac{p}{\rho} + \frac{u^2}{2}$, $i = E + \frac{p}{\rho}$, $G_{\rm m} = \rho_0 a_* F$, and $\delta q = \frac{Q}{G_{\rm m} i_0}$.

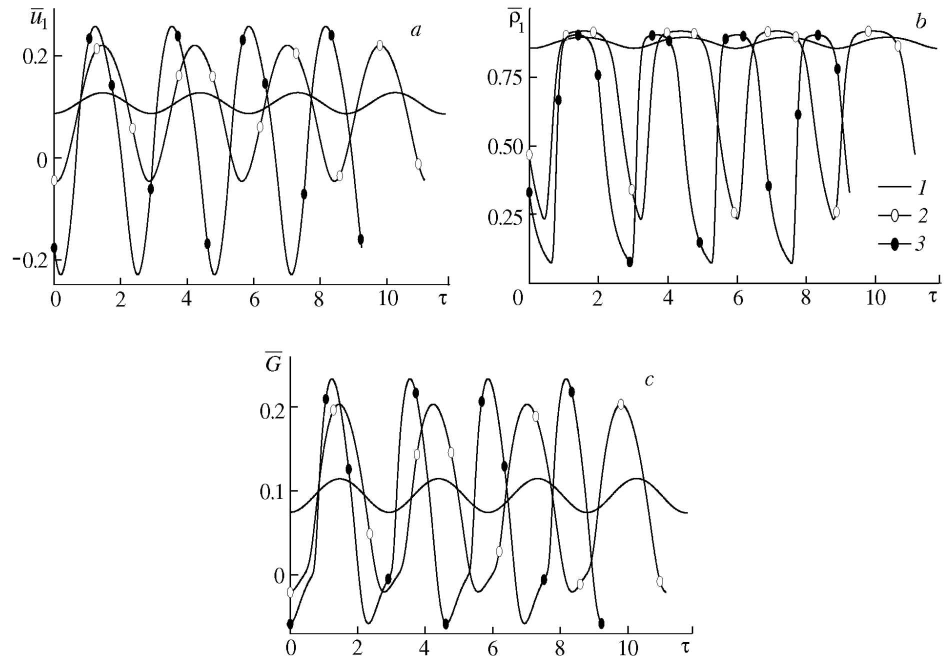

Fig. 3. Change in the dimensionless parameters of the gas flow in the inlet cross section of the pipe as a function of the dimensionless time for the self-oscillatory regime of flow at δp = 0.8 and the values of the parameter δq = 1.36 (1), 1.5 (2), and 2.0 (3): a) velocity; b) density; c) flow rate.

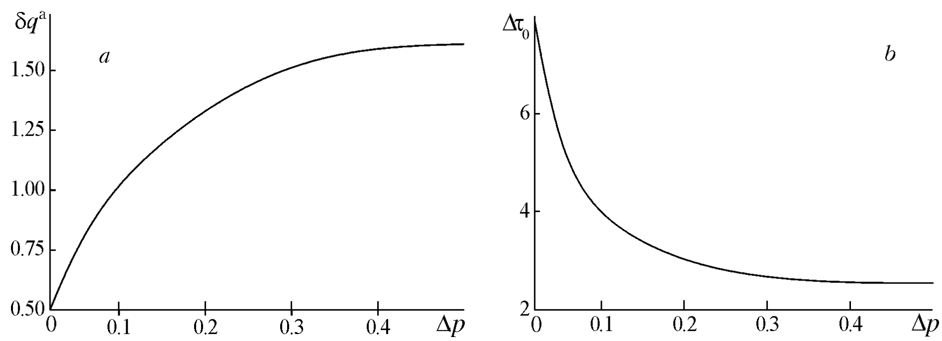

Fig. 4. Changes in the parameters of initiation of self-oscillations as functions of the parameter Δp: a) δq^{a}; b) $\Delta\tau_0$.

Equations (8) are solved numerically with the Godunov finite-difference scheme [4]. The condition of isentropic flow out of the receiver or flow into the receiver is specified as a boundary condition in the inlet pipe cross section; the parameters of the stagnant gas in the receiver are used: the pressure p_0 and the stagnation enthalpy i_0 determined from the temperature T_0 of the gas in the receiver. The condition of equality of pressures in the outlet cross section of the pipe and in the ambient space is specified. The boundary conditions mentioned are realized in accordance with [4].

The determining parameters of the considered problem of gas flow out of the receiver through the pipe are δq and δp. As the parameter δq grows, the stationary regime of outflow is realized (see Fig. 1); however, the regime of flow changes to a self-oscillatory one for δq higher than a certain δq^{a} value. As the parameter δq grows further, the oscillation amplitude increases, whereas the period decreases. This is illustrated by Fig. 3, which shows four harmonics of variation in dimensionless velocity, density, and flow rate in the inlet pipe cross section for different values of the parameter δq when δp = 0.8.

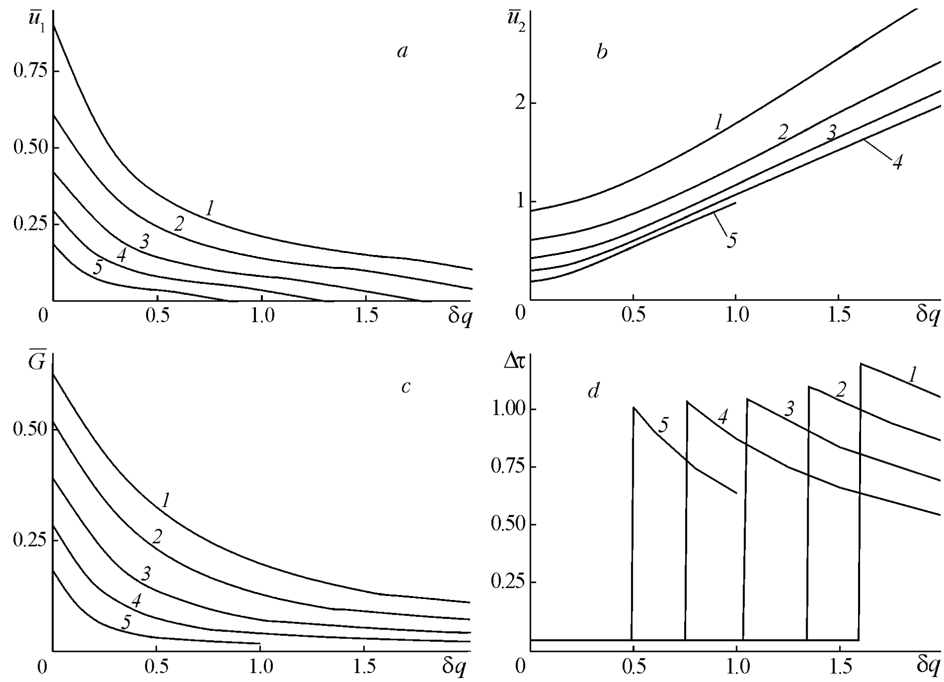

Fig. 5. Changes in the dimensionless gas parameters averaged over the oscillation period as functions the problem's parameter δq: a and b) average gas velocity at the pipe inlet and outlet; c) average flow rate of the gas at the pipe inlet; d) oscillation period; 1) Δp = 0.40, 2) 0.20, 3) 0.10, 4) 0.05, and 5) 0.2.

For interpretation of the results it is convenient to introduce the relative difference of the pressures in the receiver and the ambient medium $\Delta p = (p_0 - p_2)/p_0$ or $\Delta p = 1 - \delta p$. Figure 4a plots the parameter δq^{a} for which the self-oscillatory regime begins to be realized as a function of the relative pressure difference Δp. It is seen that the smaller the relative pressure difference Δp, the lower the value of the parameters δq^{a} for which the changeover to an unsteady self-oscillatory regime of outflow occurs. The initial period of self-oscillations $\Delta\tau_0$ as a function of the parameter Δp in changeover from the stationary regime of flow to a self-oscillatory one is plotted in Fig. 4b. It is seen that the initial period of self-oscillations grows with decrease in the pressure difference.

To analyze the parameters of outflow on changeover to self-oscillations we use the parameters averaged over the oscillation period.

Figure 5 shows the dimensionless gas velocities u_1 and u_2 averaged over the oscillation period (Fig. 5a and b), the average dimensionless gas flow rate G (Fig. 5c), and the oscillation period $\Delta\tau$ (Fig. 5d) as functions of the parameter δq for Δp = const. It follows from this figure that the gas flow is decelerated at the inlet with increase in the parameter δq and is accelerated at the pipe outlet. Further growth in δq gives rise to the average negative velocities at the pipe inlet; the average gas flow rate G in the inlet pipe cross section remains positive. The reason is that, as follows from Fig. 3a and b, when the heat supply is highly intense, the gas velocity in the inlet cross section takes negative values, i.e., the gas flows into the pipe out of the receiver at certain instants of time on the oscillation period. Since the temperature of the gas flowing into the receiver is fairly high due to its heating in the pipe, its density is much lower than the density of a cold gas flowing out of the receiver into the pipe for positive velocities, and the flow rate of the gas during its flow out of the pipe into the receiver is small. In this connection, for a fairly large heat supply, when the gas velocity (average over the oscillation period) in the inlet pipe cross section becomes negative, the average gas flow rate by virtue of the low density of the hot gas flowing into the receiver out of the pipe remains positive.

Increase in the thermal action on the subsonic flow in the pipe leads to a decrease in the average mass flow rate of the gas for the prescribed relative pressure difference Δp; the larger Δp corresponds to the larger average flow rate of the gas.

The dependence of the self-oscillation period $\Delta\tau$ on the relative pressure difference and the supplied quantity of heat can be tracked in Fig. 5d. In the stationary regime of outflow, we have $\Delta\tau = 0$. At the instant of destruction of the stationary flow and occurrence of self-oscillations, $\Delta\tau$ changes abruptly and takes the maximum value $\Delta\tau_0$. This makes it possible to determine, from Fig. 5d, the values of the parameter $\delta q = \delta q^{\mathrm{a}}$ for which self-oscillations appear at different Δp values. Further increase in $\delta q(\delta q > \delta q^{\mathrm{a}})$ leads to a decrease in the period and consequently an increase in the self-oscillation amplitude.

The results are given in Fig. 5 in the form of a two-parametric dependence. Since self-oscillations appear in the case of a fairly large heat supply, it is of interest to allow for the dependence of the outflow parameters on δp (the dependences is yielded by the above asymptotic formulas) for selection of the characteristic scale of the determined and determining quantities. Using (5) and (7), we write expressions for the scale of velocity at the pipe inlet and outlet — U_{m1} and U_{m2} — and of time:

$$U_{\mathrm{m1}} = a_* \sqrt{f(\delta p)\,(1 - f(\delta p))}\,, \tag{9}$$

$$U_{\mathrm{m2}} = a_* \sqrt{\frac{f(\delta p)}{1 - f(\delta p)}}\,, \tag{10}$$

$$t_{\mathrm{m}} = L / U_{\mathrm{m1}}\,.$$

For scaling of the gas flow rate we use the following expression:

$$G_{\mathrm{a}} = \rho_0 U_{\mathrm{m1}} F = \rho_0 a_* F \sqrt{f(\delta p)\,(1 - f(\delta p))}\,. \tag{11}$$

From the introduced scales and the above asymptotic formulas, we can write the dependences for the velocities at the pipe inlet and outlet and for the flow rate in the form

$$\bar{u}_1 = \frac{1}{\sqrt{q}}\,, \quad \bar{u}_2 = \sqrt{q}\,, \quad \bar{G} = \frac{1}{\sqrt{q}}\,. \tag{12}$$

Thus, the scales introduced enable us to represent, in large heat supply, the parameters of stationary flow as a function of only one parameter $q = Q/(G i_0)$ which is equal to the quantity of the supplied heat referred to the product of the running averaged flow rate and the stagnation enthalpy of the flowing-out gas. In this connection, to diminish the dependence of the time-averaged dimensionless parameters on δp (or Δp) in the self-oscillatory regime it is of interest to use, in addition to the scales introduced by relations (9)–(11) for the determined quantities, the parameter q related to δq by

$$q = \delta q G_{\mathrm{m}} / G\,, \tag{13}$$

where G is the time-averaged flow rate resulting from the solution of Eq. (8), as the determining parameter.

Figure 6a–c gives the time-averaged values of the gas velocity at the pipe inlet and outlet and of the flow rate as functions of q for several values of δp; they have been made dimensionless with the scales introduced. It is seen that all the curves in the figure which correspond to different pressure ratios δp are virtually coincident. This enables us to carry out calculations only for one values of the parameter δp, whereas for other δp values the solution is obtained by simple recalculation of the scale factor. The dash curve in the figure shows the results obtained from relations (12). Good agreement of the results in both the stationary regime of outflow and after the occurrence of self-oscillations is seen.

Figure 6d plots the dimensionless periods of oscillations for different Δp. It is clear that, in new dimensionless period, the range in which changeover to self-oscillations is carried out is substantially contracted compared to the changeover range in Fig. 5d.

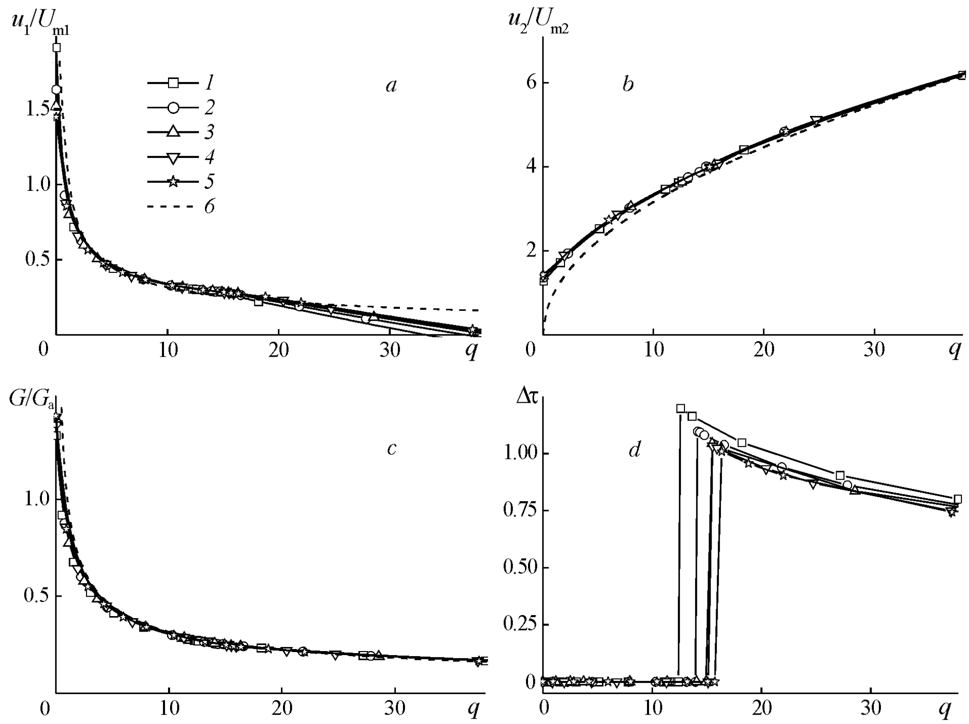

Fig. 6. Changes in the dimensionless gas parameters averaged over the oscillation period as functions of the parameter q: a) average gas velocity at the pipe inlet; b) average gas velocity at the pipe outlet; c) average flow rate of the gas at the pipe inlet; d) oscillation period; 1) Δp = 0.40, 2) 0.20, 3) 0.10, 4) 0.05, 5) 0.02, 6) asymptotic formulas.

The graphic data in Fig 6 are universal in character and can be used for a priori evaluations of the outflow parameters and the conditions of changeover to a self-oscillatory regime. Below we give the procedure for determining the values of the physical parameters of the problem for the parameter δp. For the prescribed parameter δp and parameters in the receiver p_0 and T_0, we find, using (11) and (18), the ratio of the flow-rate scale G_a to G_m:

$$\frac{G_a}{G_m} = \sqrt{f(\delta p)\,(1 - f(\delta p))}\ . \tag{14}$$

We compute ρ_0 and a_* and find the flow-rate ratio G_a/G_m from formula (14). Next, we select the value of the parameter q and take the value of $K_g = G/G_a$ from the $G(q)$ plot (see Fig. 6c). We find the parameter δq corresponding to the selected parameter q by using expression (13) rewritten in the form

$$\delta q = qK_g\,(G_a/G_m)\ . \tag{15}$$

The average physical flow rate is computed as the product of K_g and G_a. Analogously we find the average physical gas velocity at the pipe inlet and outlet with the use of the scale factors U_{m1} (9) and U_{m2} (10) respectively.

CONCLUSIONS

1. We have shown, by calculation, the self-oscillatory regimes of flow of a gas out of a vessel with prescribed stagnation parameters through a pipe (channel) to a medium with a prescribed counter pressure in the presence of intense heat supply. It has been shown that the character of these flows is determined by two parameters — the dimensionless heat supply δq and the ratio of the pressures in the ambient medium and the receiver δp. The dependences of the parameters of self-oscillations on the determining parameters of the problem have been elucidated.

2. We have obtained the asymptotic dependences for the parameters of the flow in the case of an infinitely large heat supply, which are used to obtain scale factors. It has been shown that these factors enable us to represent the two-parametric distributions of the averaged flow parameters $f(\delta q, \delta p)$ in the form of unified generalized graphic dependences $f(q)$ for the period-averaged parameters of nonstationary self-oscillating flows only on one parameter q, i.e., the supplied heat, whereas the explicit dependence of the solution on the parameter δp (pressure ratio) is excluded.

NOTATION

a, velocity of sound; a_*, critical velocity of sound in the receiver; a_{*_1} and a_{*_2}, critical velocity of sound in the inlet and outlet cross section of the pipe; E, specific total energy of the gas; F, cross-sectional area of the pipe; G, mass-mean flow rate of the gas in the inlet pipe cross section; G_a, characteristic scale of flow rate of the gas, determined with asymptotic formulas; G_m, flow rate scale determined from the parameters of the gas in the receiver; i, specific total enthalpy of the gas; i_0, specific total enthalpy of the gas in the receiver; K_g, ratio of the gas flow rate to the characteristic flow-rate scale, taken from the plot; L, pipe length; M_1, Mach number in the inlet cross section of the pipe; p, pressure; p_0, stagnation pressure in the receiver; p_1, pressure in the inlet cross section of the pipe; p_2, pressure in the outlet cross section of the pipe or in the ambient medium; q, heat supplied in a unit time to a unit mass of the gas and referred to the specific stagnation enthalpy of the gas in the receiver; Q, total heat supplied to the gas in the pipe in a unit time; R, gas constant; t, time; t_m, characteristic time scale; T_0, stagnation temperature in the receiver; u, gas velocity; u_1 and u_2, gas velocities in the inlet and outlet cross section of the pipe; U_{m1} and U_{m2}, characteristic inlet- and outlet-velocity scales determined with asymptotic formulas; x, longitudinal coordinate; δp, ratio of the ambient pressure to the stagnation pressure in the receiver; δq, parameter of thermal action on the flow; δq^a, value of the parameter δq for which changeover to self-oscillations is carried out; Δp, relative difference of the pressures in the receiver and the ambient medium; $\Delta\tau$, dimensionless self-oscillation period; $\Delta\tau_0$, dimensionless initial self-oscillation period; γ, adiabatic exponent of the gas; λ, velocity coefficient of the gas; λ_1 and λ_2, velocity coefficients of the gas in the inlet and outlet cross section of the pipe; ρ, gas density; ρ_0, stagnation density of the gas in the receiver; τ, dimensionless time. Subscripts and superscripts: a, self-oscillations (autooscillations); g, data taken from the plots; m, scale quantities; 0, parameters of stagnation in the receiver; 1, at the pipe inlet; 2, at the pipe outlet; *, critical parameters in the receiver; $^{-}$, dimensionless parameters.

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